allowance for the term P_4 makes a contribution to the pressure distribution no larger than $O(\tau^{4-2/\kappa}\zeta)$. In the present report neither of these corrections was taken into account in the derivation of the expression for x_1. The form of the remaining gasdynamic quantities u_1 and u_2 is fully determined by the distributions of the functions x_2, p, and ρ, and therefore, no comparison is made for them.

In the general case the agreement established above allows one to effectively construct a solution to the inverse steady problem any time that a solution is constructed in the corresponding unsteady one-dimensional problem.

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SEMIEMPIRICAL THEORY OF THE GENERATION OF DISCRETE TONES BY A SUPERSONIC UNDEREXPANDED JET FLOWING OVER AN OBSTACLE

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§1. The phenomenon of the generation of a strong discrete tone by a supersonic underexpanded jet flowing over an obstacle was first discovered by Hartmann [1]. There are presently a considerable number of reports devoted to the experimental study of the Hartmann effect [2-4]. However, the mechanism of formation of these oscillations has not been clarified up to now [2, 5]. An elementary theory of this phenomenon is presented in the present report.

A diagram of a supersonic underexpanded jet flowing over a flat obstacle is presented in Fig. 1 (1 is the jet boundary; 2 is the central compression shock (the Mach disk); 3 is the suspended shock; 4 is the reflected shock; and 5 is the contact discontinuity). The effect consists in the fact that the flow becomes unstable at certain values of the nozzle Mach number M_a, degree of nonratedness $n = p_a/p_s$ of the jet (p_a is the pressure in the jet

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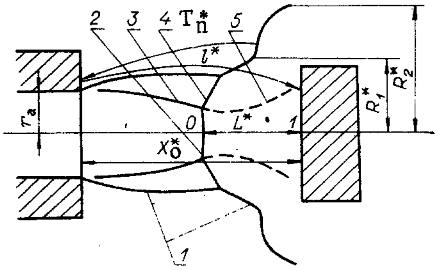

Fig. 1

in the plane of the nozzle cut and p_s is the pressure in the space surrounding the jet), and distance $X_o = X_o^*/r_a$ and size $r_o = r_o^*/r_a$ of the obstacle. The jet boundary and the wave structure contain periodic oscillations. In this case the level of the pressure oscillation at the obstacle can reach 190-200 dB and in the nearby acoustic field, 170-180 dB.

The following mechanism for the formation of the oscillations is proposed on the basis of experimental observations. The process develops from small perturbations owing to the loss of flow stability [6]. In this case the resonator is the region of subsonic flow between the Mach disk and the obstacle. The sound waves emitted from the resonator into the outer space reach the nozzle cut and give rise to a perturbation at the jet boundary. In propagating from the nozzle to the obstacle the perturbations grow in intensity because of the instability of the jet boundary as a tangential discontinuity. Upon reaching the surface of the obstacle they are stopped, which gives rise to pressure pulsations at the obstacle. The interaction of the latter with the oscillations in the resonator at suitable amplitude—phase ratios leads to the formation of self-oscillations in the system under consideration. The given scheme of the phenomenon differs from Morch's scheme [7], which was proposed earlier and did not lead to success, by the introduction of feedback through the outer acoustic field [6].

§2. We will present a solution which allows one to determine the regions and frequencies of the oscillations. The oscillations in the resonator are assumed to be one-dimensional, since according to a motion picture the Mach disk undergoes plane oscillations. To simplify the calculations the Mach number of the flow up to the obstacle is taken as constant ($M \approx 0.5\ M_2$), since not allowing for the distribution of M in the resonator leads to an error of no more than ≈ 5% in the determination of the frequency. With these assumptions the small perturbations in the resonator can be described by the system of equations [8]

$$\begin{gathered}\partial v/\partial\tau + M\partial v/\partial\xi + \partial p/\partial\xi = 0;\\ \partial p/\partial\tau + M\partial p/\partial\xi + \partial v/\partial\xi = 0,\end{gathered} \tag{2.1}$$

where $p = \delta p/kp^*$ and $v = \delta v/a$ are the dimensionless perturbations in pressure and velocity; k is the adiabatic index; p^* and a are the average pressure and the velocity of sound; $\xi = x^*/L^*$ and $\tau = at/L^*$ are the dimensionless coordinate and dimensionless time; L^* is the length of the resonator.

The solution of system (2.1) is represented in the form

$$\begin{aligned}p &= 0.5\left[A_p\left(\exp\left\{-\frac{\beta\xi}{M+1}\right\} + \exp\left\{-\frac{\beta\xi}{M-1}\right\}\right) + A_v\left(\exp\left\{-\frac{\beta\xi}{M+1}\right\} - \exp\left\{-\frac{\beta\xi}{M-1}\right\}\right)\right]\exp\{\beta\tau\};\\ v &= 0.5\left[A_v\left(\exp\left\{-\frac{\beta\xi}{M+1}\right\} + \exp\left\{-\frac{\beta\xi}{M-1}\right\}\right) + A_p\left(\exp\left\{-\frac{\beta\xi}{M+1}\right\} - \exp\left\{-\frac{\beta\xi}{M-1}\right\}\right)\right]\exp\{\beta\tau\},\end{aligned} \tag{2.2}$$

where $\beta = \nu + i\omega$ is the dimensionless frequency of the oscillations; $\omega = 2\pi f L^*/a$; M is the average Mach number on the resonator axis.

Thus, the flow in the resonator is assumed to be one-dimensional flow having a constant axial velocity.

The boundary condition must be formulated in the cross sections $\xi = 0$ and $\xi = 1$. In the cross section $\xi = 0$ it can be written in the form

$$v + \lambda_1 p = 0. \tag{2.3}$$

The acoustic conductance λ_1 of the straight compression shock is determined [9] by the expression

$$\lambda_1 = \frac{M_1^2 + 1}{2M_1^2 M_2},$$

where M_1 and M_2 are the Mach numbers in front of the central compression shock and behind it, respectively.

Along the boundary of the jet the stream moves with a constant [10] Mach number

$$M_s = \sqrt{\frac{2}{k-1}\left[\left(1 + \frac{k-1}{2} M_a^2\right) n^{\frac{k-1}{k}} - 1\right.}.$$

In this case the pulsations in the pressure $p(\xi = 1)$ at the obstacle can be connected with the pulsations in sonic pressure p_n at the nozzle face as follows:

$$p(\xi = 1) = p_n \exp\{\varphi_r + i\varphi_i\}. \tag{2.4}$$

Here φ_r depends on the degree of growth in the amplitude of a perturbation during the propagation of the latter downstream along the jet boundary from the processes of refraction of the sound wave within the jet and of transformation of the perturbation of the obstacle. The quantity φ_i represents the phase shift between the pressure oscillation at the nozzle face and the pressure oscillation at the obstacle.

Equation (2.4) is the boundary condition at $\xi = 1$, in which one must still determine the quantities p_n, φ_r, and φ_i.

§3. It follows from experiment that in the outer space the acoustic waves are emitted by part of the boundary of the jet reflected from the obstacle, namely, by an annular surface of width $(R_2^* - R_1^*)$ (see Fig. 1). Using the method applied in [11] to calculate the acoustic field of the piston diaphragm, one can write the sound pressure at the face in the form

$$p_n = N \frac{\beta \exp\left\{-\beta \frac{r_n}{L}\right\}}{2r_n L} (R_2^2 - R_1^2), \tag{3.1}$$

where

$$r_n = r_n^*/r_a; \quad L = L^*/r_a; \quad R_1 = R_1^*/r_a; \quad R_2 = R_2^*/r_a; \quad N = N^*/a_0$$

(a_0 is the velocity of sound in the space surrounding the jet; N^* is the velocity of the motion of the emitting surface). In the derivation of Eq. (3.1) it was assumed that the emitting surface is flat, it moves parallel to itself, and the jet does not affect the range of the sound beams. This is valid if the transverse size of the jet is less than the emitted wavelength. An estimate of the diffraction of the emitted sound wave on the jet as on an obstacle, made using [12], showed that the perturbing effect of the jet on the acoustic field can be neglected when the direction of propagation of the sound wave in the outer field is along the boundary of the incident jet. The nozzle face has a size much smaller than the wavelength of the oscillations, and therefore it is considered as sharp, i.e., reflection of the wave from the nozzle is absent.

In an experimental test of Eq. (3.1) it was established that it satisfactorily describes the outer acoustic field at a point near the nozzle cut. Experiments using high-speed motion-picture photography showed that the emitting surface and the Mach disk oscillate in phase and with the same amplitude, i.e., N^* is the velocity of oscillation of the Mach disk. A comparison of the values of p_n measured by a pressure pickup at the nozzle face with p_n calculated by Eq. (3.1) showed a difference not exceeding 10-15% (depending on the mode of flow). In these calculations the R_2 and N entering into Eq. (3.1) were determined experimentally.

As shown in [9], the following equation is valid at a straight compression shock:

$$N/p(\xi=0)=(k+1)/4\mathrm{M}_2. \tag{3.2}$$

Therefore, the boundary condition (2.4) at $\xi = 1$, with allowance for (3.1) and (3.2), can be written in the form

$$p(\xi=1)=p(\xi=0)\frac{k+1}{4\mathrm{M}_2}\frac{\beta\exp\left\{-\beta\frac{r_n}{L}\right\}}{2r_n L}(R_2^2-R_1^2)\exp\{\varphi_r+i\varphi_i\}. \tag{3.3}$$

§4. We express φ_r and φ_i through the parameters of the steady stream. Keeping in mind that the basic processes characterizing the development of perturbations in a free jet and a jet with an obstacle are identical, one can write

$$\varphi_r=F_1\varphi_r'; \quad \varphi_i=F_2\varphi_i', \tag{4.1}$$

where φ_r' and φ_i' are the growth coefficient and the phase shift for a free jet; F_1 and F_2 are quantities which depend on the processes of transformation of the perturbations at the nozzle face and at the obstacle. The results of [13], devoted to the stability of a free homogeneous supersonic jet, are used to seek the expressions for φ_r' and φ_i'.

Graphs of the dependence of the real and imaginary parts of the complex quantity $k'v^*/\omega^*$ on the Strouhal number $\mathrm{Sh} = 2r_a f/a_0 \mathrm{M}_s$ for $\mathrm{M}_s = 1.2$–2.4 are presented in the cited report. Here $k' = k_r' + ik_i'$ is the complex wave number; f is the frequency; $\omega^* = 2\pi f$; v^* is the stream velocity along the jet boundary. Since φ_r' is a function of the imaginary part, while φ_i' is a function of the real part of $k'v^*/\omega^*$, by approximating the functions (presented in [13]) by simple equations (for $\mathrm{Sh} < 0.2$) one can obtain

$$\varphi_r'=3\pi\sqrt{1+\frac{k-1}{2}\mathrm{M}_s^2}(\mathrm{Sh})^2 l; \quad \varphi_i'=\pi\sqrt{1+\frac{k-1}{2}\mathrm{M}_s^2}\ \mathrm{Sh}\,[1+(\mathrm{M}_s-1{,}2)\mathrm{Sh}]\,l. \tag{4.2}$$

On the basis of the experimental results presented in [14] one can compare the values of φ_r' and φ_i' calculated from Eqs. (4.2) with the experimentally measured values. However, instead of φ_i' it is more convenient to compare the wavelength $\lambda = v^*/f\mathrm{Re}\{k'v^*/\omega^*\}$. For example, for $r_a = 2$ cm, $\mathrm{Sh} = 0.18$, $\mathrm{M}_s = 1.73$, and antisymmetric oscillations ($n = 1$ in the notation of [13]) the following values are obtained: measured $\lambda = 12.0$ cm, calculated $\lambda = 15.2$ cm; measured $\varphi_r'^* = 0.286\ \mathrm{cm}^{-1}$, calculated $\varphi_r'^* = 1.00\ \mathrm{cm}^{-1}$. It follows from the comparison that the results of [13] do not adequately take into account the true dependence of φ_r' and φ_i' on the steady parameters of the jet. One can therefore expect that the noted noncorrespondence between the calculated and measured values of φ_r' and φ_i' depends systematically on the cited parameters. Consequently, the quantities F_1 and F_2, determined experimentally below, must take into account the noted noncorrespondence in addition to the transformations of the perturbations indicated earlier.

The quantity F_1 was determined by the following method: pressure pickups were mounted at the nozzle face and at the center of the obstacle. The ratio of amplitudes $|p(\xi = 1)|/|p_n| = e^{\varphi_r}$ of the measured pressures was calculated for the modes of flow with self-oscillations and the value of φ_r was calculated. The value of φ_r' was calculated from Eq. (4.2), with the value of the frequency which enters into the Strouhal number being determined from experiment. It was found that for the different modes of flow the ratio of the measured value φ_r to the calculated value φ_r' remains unity with an acceptable accuracy if the calculated value φ_r' is multiplied by the function $F_1 = \mathrm{M}_s L$.

Following the substitution of the value of F_1 into (4.1) and transformations, we obtain the following expression for φ_r:

$$\varphi_r=0.955\sqrt{1+\frac{k-1}{2}\mathrm{M}_s^2\frac{\omega^2}{L\mathrm{M}_s}}\,l. \tag{4.3}$$

To find F_2 the phase velocity was determined (using high-speed motion-picture photography and using a thermoanemometer). It was established that $F_2 = 1.25$. As seen from the example presented above for a free jet ($r_a = 2$ cm, $\mathrm{Sh} = 0.18$, $\mathrm{M}_s = 1.73$), the ratio of the calculated to the measured λ is also close to 1.25.

Following the substitution of the value of F_2 into the expression for φ^i (4.1) and transformations, we finally obtain

$$\varphi^i = 1.25 \sqrt{1+\frac{k-1}{2}M_s^2}\,\frac{\omega}{M_s L}\left(1+\frac{M_s-1.2}{\pi M_s L}\right)l. \tag{4.4}$$

Thus, all the influences on the evolution of a perturbation from the nozzle face to the obstacle are taken into account empirically in Eqs. (4.3) and (4.4).

§5. From Eqs. (2.2), (2,3), and (3.3) one can obtain the characteristic equation for the calculation of the regions and frequencies of the self-oscillations in the Hartmann effect. In the case of $M_1 \geqslant 3.0$, which usually occurs in the phenomenon under investigation, the following inequality is satisfied:

$$|(1-\lambda_1)\exp\{-\beta/(M+1)\}| \ll |(1+\lambda_1)\exp\{-\beta/(M-1)\}|.$$

With this condition the characteristic equation has the form

$$\frac{k+1}{4M_2}\beta\frac{\exp\left\{-\beta\frac{r_n}{L}\right\}}{r_n L}(R_2^2-R_1^2)\exp\{\varphi_r+i\varphi_i\}\approx(1+\lambda_1)\exp\left\{-\frac{\beta}{M-1}\right\}.$$

We introduce the designations

$$\psi_1 = \frac{k+1}{4M_2}\frac{(R_2^2-R_1^2)}{r_n L(1+\lambda_1)},\quad \psi_2=\frac{r_n}{L}+\frac{1}{1-M}$$

and by separating the real and imaginary parts we obtain a system of two equations for the calculation of ν and ω:

$$\begin{aligned}\exp\{\nu\psi_2\}\cos\psi_2\omega &= \psi_1\exp\{\varphi_r\}(\nu\cos\varphi_i-\omega\sin\varphi_i);\\ \exp\{\nu\psi_2\}\sin\psi_2\omega &= \psi_1\exp\{\varphi_r\}(\omega\cos\varphi_i+\nu\sin\varphi_i).\end{aligned} \tag{5.1}$$

For convenience in calculating the roots one can reduce system (5.1) to the form

$$\begin{aligned}\nu &= \omega/\mathrm{tg}\,(\psi_2\omega-\varphi_i),\\ \exp\{2\psi_2\nu\} &= \psi_1^2\exp\{2\varphi_r\}(\nu^2+\omega^2).\end{aligned} \tag{5.2}$$

In this case system (5.2) can have roots which are not roots of the original system (5.1). If as a result of the solution of system (5.1) it turns out that $\nu < 0$, then the jet flow is stable, while if $\nu > 0$, then self-oscillations should develop.

The coefficients in Eqs. (5.1) and (5.2) depend on the average parameters of the jet, and the latter are assumed to be known.

§6. Let us compare the results of a calculation carried out with the solution of system (5.2) with published experimental data. To determine the steady parameters of the jet required in the calculations we used the empirical functions presented in [15-17]. As a result of a study of a large number of photographs of the shadow pattern of the flow and the wave pattern of the emitted acoustic field for jets with the parameters M_a = 1-2.0, and n = 1-20, and X_o = 4-14 it was assumed that $R_2 = R_1$ for a finite obstacle if the radius of the Mach disk is larger than or equal to the radius of the obstacle. If the radius of the obstacle is larger than the radius of the Mach disk but smaller than $(3/2)R_1$, then $R_2 = 2r_o$. If $r_o > (3/2)R_1$, then $R_2 = 2R_1$ when $R_1 \leqslant 0.5l$ or $R_2 = 0.5l$ when $R_1 > 0.5l$. In the latter case the meaning of the assignment of the quantity R_2 is connected with the fact that the width of the annulus of the emitter must not be larger than one fourth the wavelength of the radiation.

The curve of neutral stability for M_a = 1.5 and r_o = 1.25 is presented in Fig. 2. The curve obtained by calculation is drawn with a solid line. The experimental points are borrowed from [3]. The curve of neutral stability for M_a = 2.0 and an infinite obstacle is presented in Fig. 3. The solid line is obtained by calculation and the dashed line is obtained experimentally [4]. The dependence of the dimensional frequency of the oscillation on the distance between the nozzle and the obstacle is presented in Fig. 4 for an infinite obstacle,

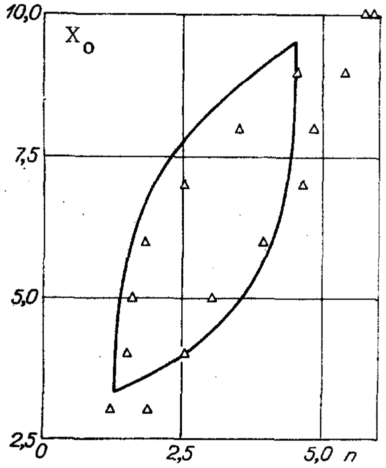

Fig. 2

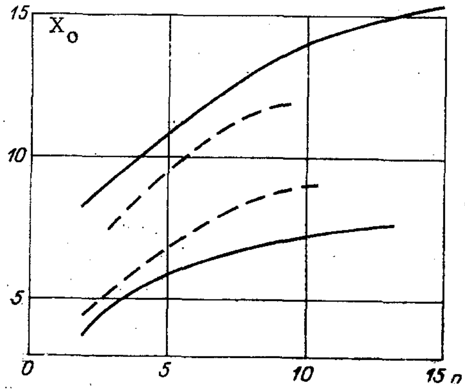

Fig. 3

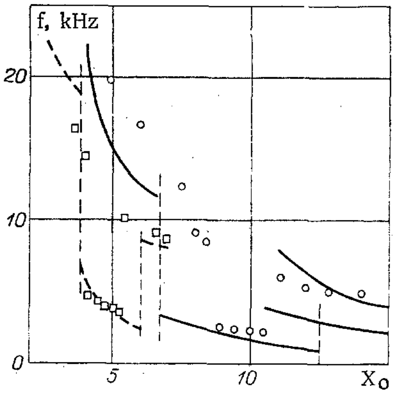

Fig. 4

a Mach number $M_a = 2.0$, and two values of the nonratedness of the jet $n = 2.0$ (dashed curve) and $n = 7.55$ (solid curve). The experimental points (squares for $n = 2.0$ and circles for $n = 7.55$) are taken from [18]. It follows from a comparison of the calculated and experimental data that the theory presented correctly reflects the basic properties of the phenomenon under study and gives satisfactory quantitative agreement with experiment. Thus, it is seen from Fig. 4 that as the obstacle gets farther from the nozzle face (X_o grows) and with a fixed nonratedness (let $n = 7.55$), self-oscillations of high frequency but, as shown by experiment, low intensity develop at a certain value of X_o. The frequency of these oscillations declines monotonically with an increase in X_o. At a certain value of X_o the frequency of the oscillations decreases abruptly and, as shown by experiment, the amplitude increases sharply. Such oscillations have come to be called the Hartmann effect. With a further increase in X_o the frequency of the oscillations declines monotonically and the amplitude decreases. Succeeding modes are then added to the fundamental mode and in certain sections one can detect two oscillation modes. With a further increase in X_o the zeroth mode disappears and the second mode continues to sound. The first mode was not detected in the experiment, although the calculation shows the possibility of its existence (but the calculation cannot indicate the intensity of the possible oscillation). For $n = 2$ the experiment reveals the zeroth and first modes, as seen from Fig. 4.

It is evident that the boundaries of the region of self-oscillations presented in Figs. 2 and 3 are not lines separating the zones of the presence and absence of oscillations having a discrete tone. They only distinguish the region of existence of the zeroth mode of oscillations which has the highest intensity.

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SOME SIMILARITY PROBLEMS OF THE UNSTEADY BOUNDARY LAYER

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UDC 533.6

It was shown in [1] that problems of the nonstationary boundary layer are similarity problems for impulsive motion of an incompressible fluid and motions accelerating with a power law. Some results for an incompressible fluid are presented below.

§1. We consider motion of a semiinfinite flat plate in a compressible liquid, impulsively set into motion. The system of equations for this case [2] is as follows:

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right);$$
$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0;$$
$$\rho\left(\frac{\partial h}{\partial t}+u\frac{\partial h}{\partial x}+v\frac{\partial h}{\partial y}\right)=\mu\left(\frac{\partial u}{\partial y}\right)^2+\frac{\partial}{\partial y}\left(\lambda\frac{\partial h}{\partial y}\right);$$
$$u=U_e,\ v=0\quad\text{for}\quad y=0,\ t=0;$$
$$u=0,\ v=0\quad\text{for}\quad y=0,\ t>0;$$
$$u=U_e,\quad h=h_e,\quad y\to\infty.$$

The notation is conventional; the viscosity μ, the thermal conductivity λ, and the equation of state are all arbitrary functions of temperature and density. We choose

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